m_π and N_c dependence of resonances from unitarized Chiral Perturbation Theory

G. Ríos

In collaboration with C. Hanhart and J. R. Peláez

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Outline

Unitarized Chiral Perurbation Theory

- Inverse Amplitude Method
- Generation of resonances

Leading $1/N_c$ behavior of the ρ and σ mesons

Chiral extrapolation of the ρ and σ mesons

Chiral Perturbation Theory

Weinberg, Gasser & Leutwyler

π's Goldstone Bosons
of the spontaneous
chiral symmetry breaking
$SU(2)_L \times SU(2)_R \to SU(2)_V$

QCD degrees of freedom
at **low energies << 4πf~1 GeV**

ChPT is the most general expansion in energies
of a lagrangian made only of pions
compatible with the QCD symmetry breaking

Leading order parameters: f_π , m_π

At higher orders, QCD
dynamics encoded in
Low Energy Constants
determined from experiment

ππ scattering
$O(p^4)$: $l_1, \cdots, l_4$
$O(p^6)$: $r_1, \cdots, r_6$

Their N_c scaling known from QCD. They are m_π independent

ChPT is the QCD Effective Theory
but is limited to low energies

Inverse Amplitude Method

Truong, Dobado, Herrero, Peláez

The (elastic) Inverse Amplitude Method (IAM) uses elastic unitarity and ChPT to evaluate a ***dispersion relation*** for 1 / t

Unitarity condition
$s \in (s_{th}, \infty)$

$$\mathrm{Im}\, t = \sigma |t|^2 \Longrightarrow \mathrm{Im}\, \frac{1}{t} = -\sigma$$

Imaginary part of 1/t known ***exactly*** on RC

ChPT, being an expansion $t \simeq t_2 + t_4 + t_6 + \cdots$
satisfies unitarity only perturbatively

$$\mathrm{Im}\, t_2 = 0$$

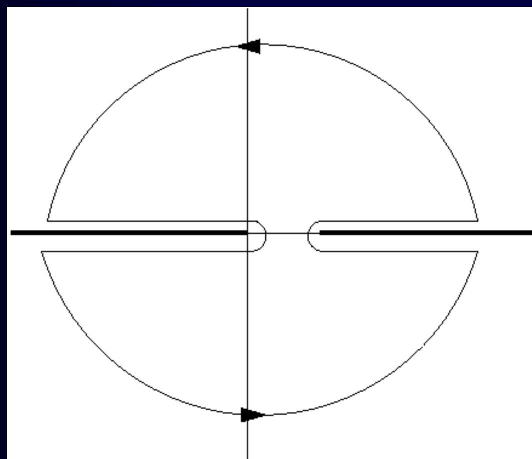
$$\mathrm{Im}\, t_4 = \sigma t_2^2$$

$$\mathrm{Im}\, t_6 = 2\sigma t_2 \mathrm{Re}\, t_4$$

$$\vdots$$

Inverse Amplitude Method

The analytic structure of 1/t (right cut, left cut and possible poles) allows us to write a dispersion relation for $G = t_2^2/t$



$$G(s) = G(0) + G'(0)s + \frac{1}{2}G''(0)s^2 + \frac{s^3}{\pi}\int_{RC}\frac{\mathrm{Im}\,G(s')\,ds'}{s'^3(s'-s-i\epsilon)} + LC(G) + PC$$

$$t_2(0) + t_2'(0)s \qquad -t_4(0) - t_4'(0)s - \frac{1}{2}t_4''(0)s^2 + \frac{s^3}{\pi}\int_{RC}\frac{-\mathrm{Im}\,t_4(s')\,ds'}{s'^3(s'-s-i\epsilon)} + LC(t_4)$$

$$t \simeq \frac{t_2^2}{t_2 - t_4}$$

This is exactly t_2

Exact dispersion relation for $-t_4$

On the right cut we use elastic unitarity

$$\mathrm{Im}\,G = -\sigma t_2^2 = -\mathrm{Im}\,t_4$$

Right cut kown **exactly** (elastic approx.)

substraction constants use ChPT:

$$G(0) \simeq t_2(0) - t_4(0)$$
$$G'(0) \simeq t_2'(0) - t_4'(0)$$
$$G''(0) \simeq -t_4''(0)$$

Calculated at a low energy point: **good approximation**

On the left cut we use ChPT:

$$\mathrm{Im}\,G \simeq -\mathrm{Im}\,t_4$$

Left cut weighted at low energies, where **ChPT valid**

PC is $O(p^6)$. We neglect it

It is small **for all s**

Inverse Amplitude Method

The IAM can be systematiclly improved taking into account higher chiral terms

At $O(p^6)$ the IAM reads:

$$t \simeq \frac{t_2^2}{t_2 - t_4 + \frac{t_4^2}{t_2} - t_6}$$

Inverse Amplitude Method

The IAM satisfies exact unitarity and matches the chiral expansion when reexpanding at low energies

Describes data up to ~ 1 GeV

Generates poles on the second Riemann sheet associated to resonances. In $\pi\pi$ scattering we find the ρ and the σ

Derived from analyticity, unitarity and ChPT.

No model dependencies, just aproximations. Use of ChPT perfectly justified, always used at low energies

Left cut and substraction constants correct up to the given ChPT order used.

Inverse Amplitude Method

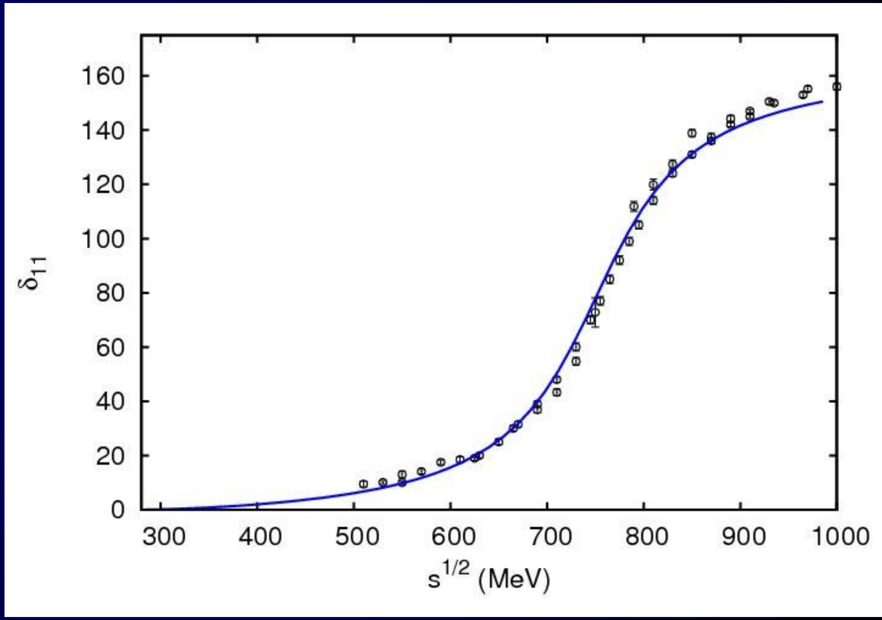
Fit pion scattering data (μ=770 MeV)

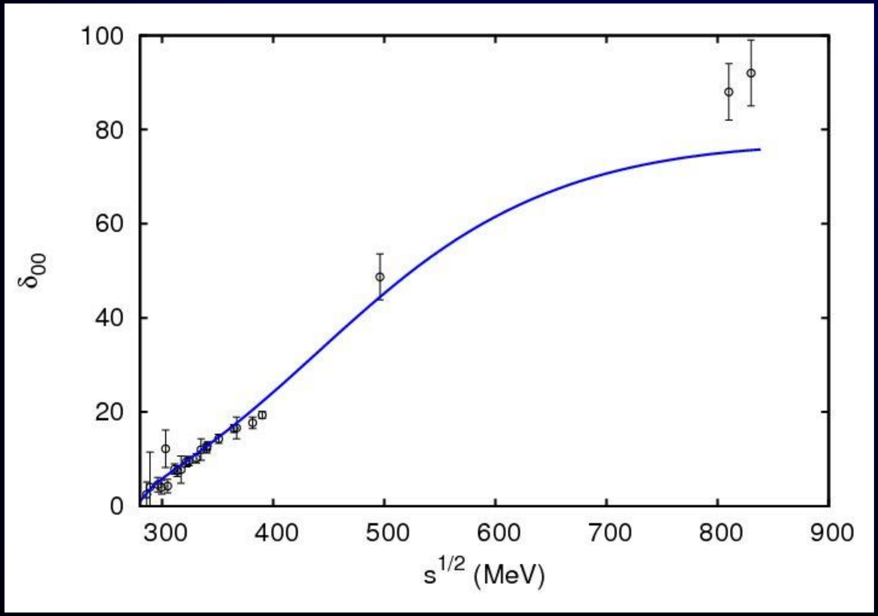
Only fit l_1 and l_2

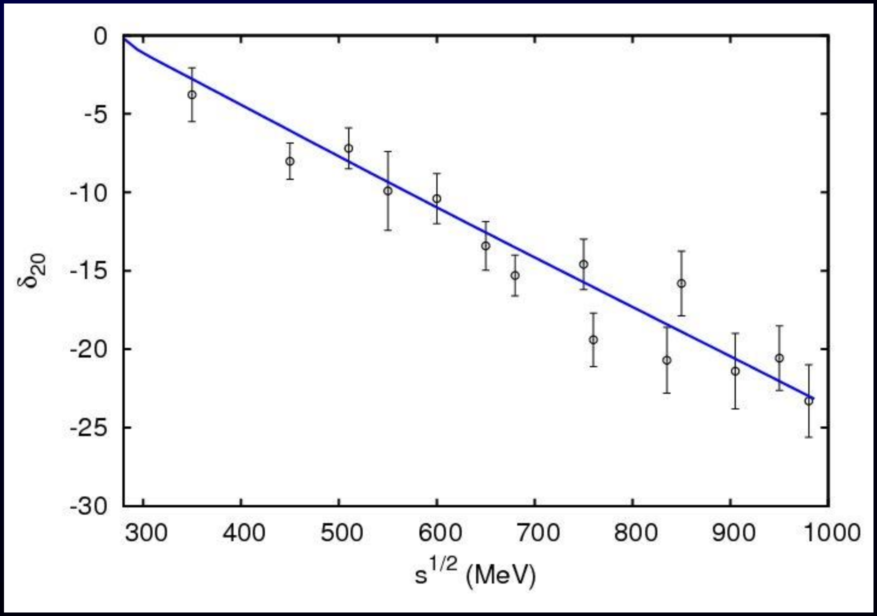
$l^r_1 = -3.7 \pm 0.2$
$l^r_2 = 5.0 \pm 0.4$

For l_3 and l_4 we take the values of Gasser and Leutwyler

$l^r_3 = 0.8 \pm 3.8$
$l^r_4 = 6.2 \pm 5.7$

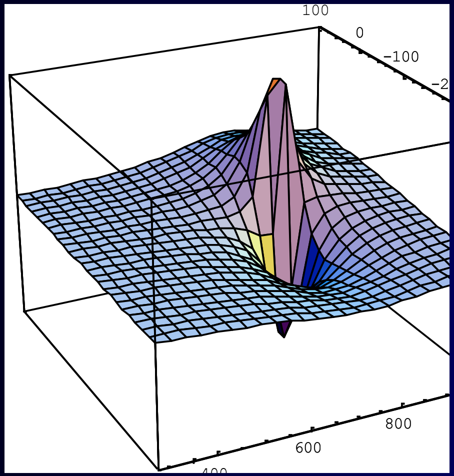


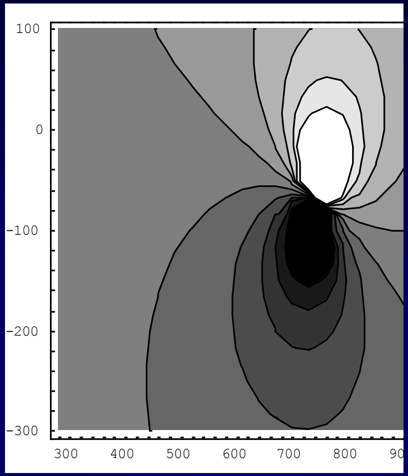




Inverse Amplitude Method

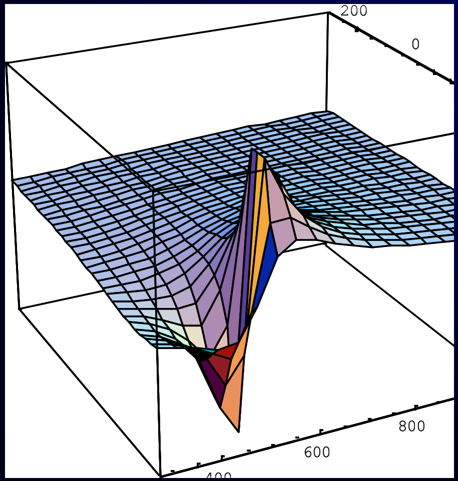
$M_\rho \sim 750$ MeV, $\Gamma_\rho \sim 150$ Mev

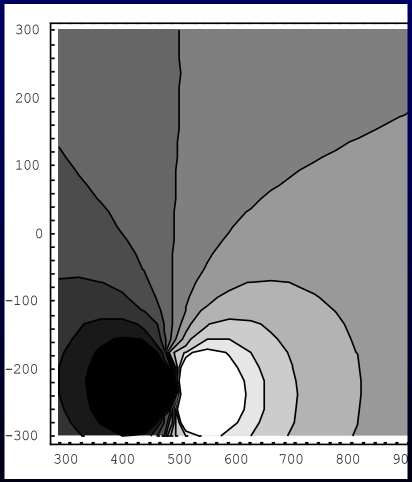




Im t_{11}

$M_\sigma \sim 440$ MeV, $\Gamma_\sigma \sim 440$ MeV





Im t_{00}

$$\sqrt{s_{pole}} = M - i\Gamma/2$$

Generates the ρ and σ resonances as poles in the 2nd Riemann sheet **without a priori assumptions on their existence and nature**

Changing parameters in the amplitudes we can study how the generated poles evolve:

- Leading $1/N_c$ behavior
- Chiral extrapolation

- Leading $1/N_c$ behavior

The 1/N_c expansion

't Hooft, Witten

The 1/N_c expansion provides a clear definition of $\bar{q}q$ states

Their masses and widths scale as

$$M_{\bar{q}q} = O(1)$$
$$\Gamma_{\bar{q}q} = O(1/N_c)$$

The QCD N_c dependence is implemented in ChPT through the LECs

$$m_\pi = O(1) \quad f_\pi = O(\sqrt{N_c})$$
$$l_i = O(N_c)$$
$$r_i = O(N_c^2)$$

We can look for the IAM pole at different N_c's and see the resonance mass and width N_c scaling and compare to that of $\bar{q}q$

$$M^{IAM}(N_c) \sim 1 \ ?$$
$$\Gamma^{IAM}(N_c) \sim 1/N_c \ ?$$

The $1/N_c$ expansion

This is only relevant ***near $N_c = 3$***

This is what gives information about the ***dominant component*** of the $N_c = 3$ physical state

Even a tiny admixture with $\bar{q}q$ will become dominant for large N_c

We dont expect the IAM to work well for very large N_c

Weak interacting limit → dispersion relation not dominated by the *exactly known* right cut

RC $1/N_c$ suppresed. At N_c=3 the IAM describes data and resonances within 10-20% errors → 100% error at $N_c \sim$ 15-30

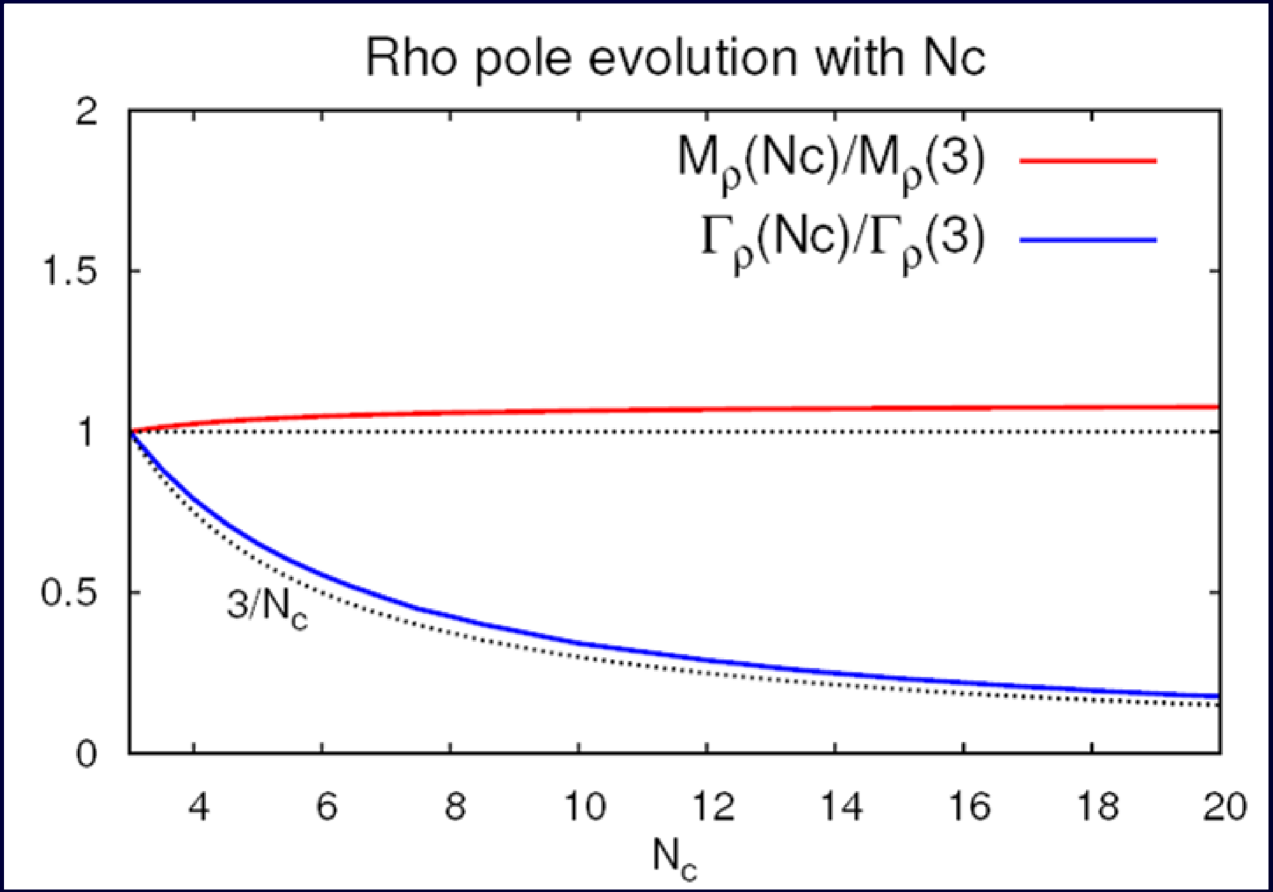
For $N_c \to \infty$ the η' also becomes a Goldstone Boson

$M_{\eta'} \sim \sqrt{3/N_c}$ → for $N_c < 30$, $M_{\eta'} \gtrsim 310$ MeV

We estimate the IAM to give reasonable results for **N_c < 15 – 30** at most

The $1/N_c$ expansion – The ρ

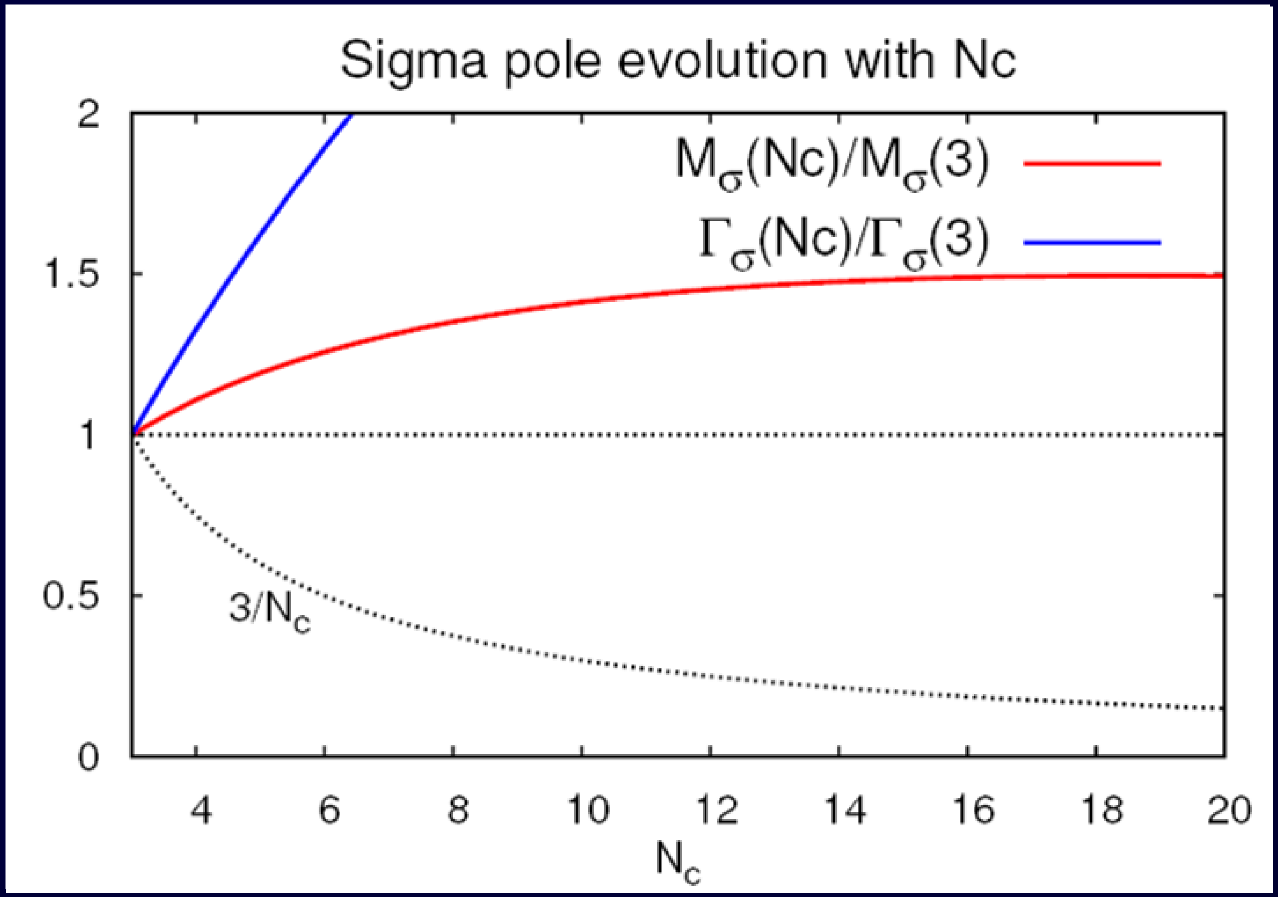
$\bar{q}q$ states: $M \sim 1, \quad \Gamma \sim 1/N_c$



The IAM generates the expected qqbar N_c scaling for the ρ

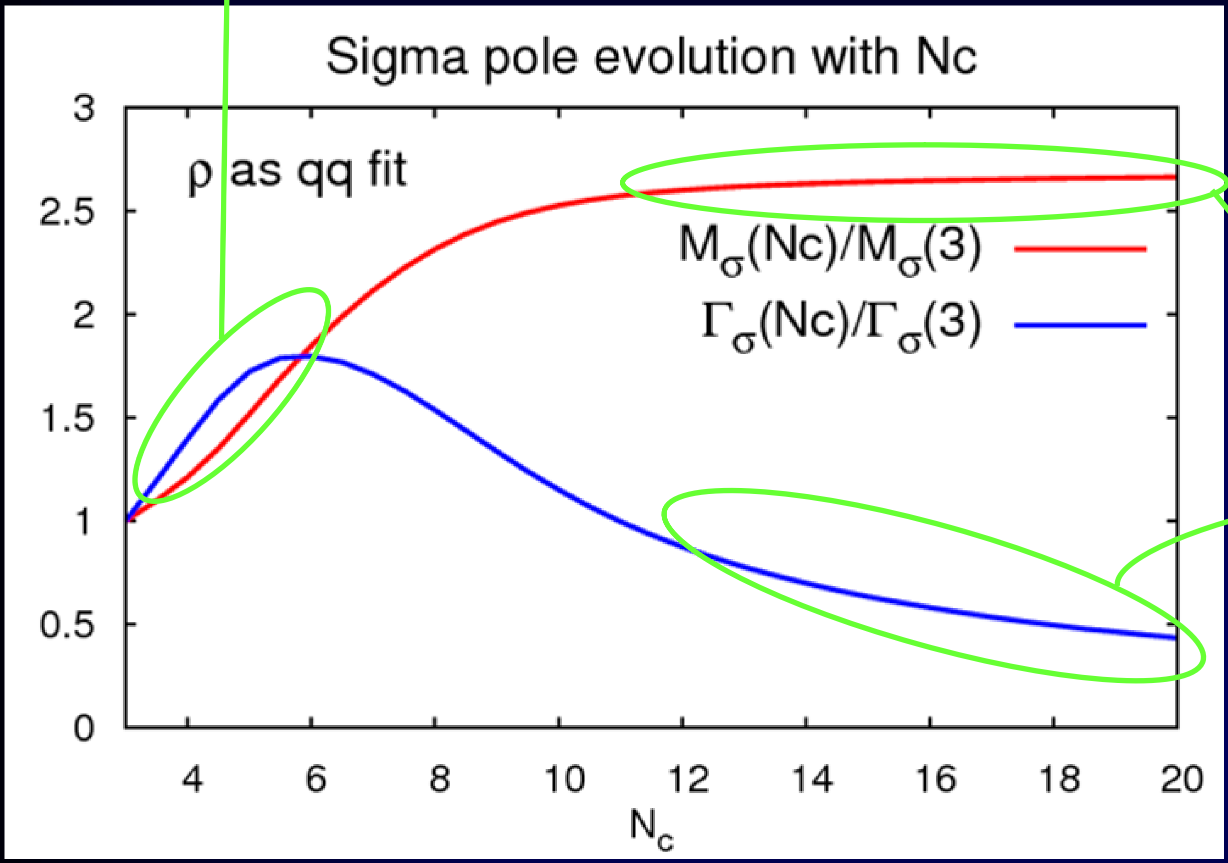
The 1/N_c expansion – The σ

$\bar{q}q$ states: $M \sim 1, \quad \Gamma \sim 1/N_c$



The sigma dominant component is not $\bar{q}q$

The 1/N_c expansion – The σ (2 loops analysis)



Near $N_c = 3$ we obtain similar results that those at $O(p^4)$

quark-antiquark subdominant component may emerge at larger Nc but at M≈1 GeV

The $f_0(600)$ still does NOT behave DOMINANTLY as quark-antiquark
(we have tried to force it but it is not possible without spoiling the rho or data)

BUT, from Nc>8 or 10, the $f_0(600)$ we might be seeing
a quark-antiquark **subdominant** component whose large Nc mass is ≥ 1 GeV

The $1/N_c$ expansion

This results suggest what the σ is NOT predominantly made of

The sigma DOMINANT component
is NOT a quark-antiquark state
tetraquark/molecule could be consistent
but glueball componnet could also contribute

Results consistent at higher orders.
The σ cannot be forced to behave as a quark-antiquark.

At two loops, a subdominant quark-antiquark component emerges above 1 GeV!!
(consistent with two nonet picture, the lightest non-quark antiquark)

- Chiral Extrapolation

Chiral extrapolation

Motivation

- LATTICE provides rigorous and systematic QCD results using quarks and gluons. Growing interest in scattering and the scalar sector.

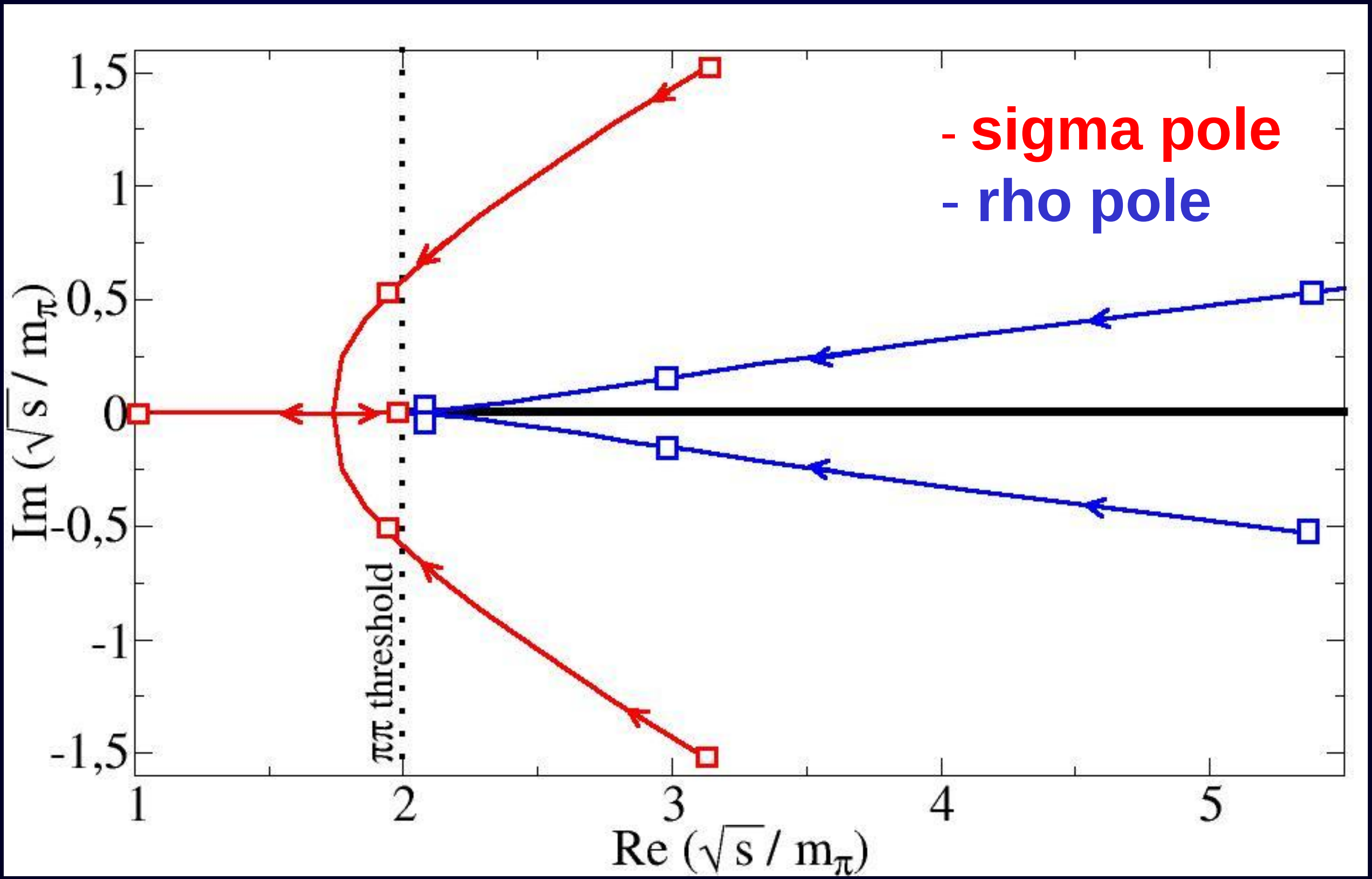
  Caveat: small, realistic, quark masses are hard to implement.

ChPT provides the correct QCD dependence of quark masses as an expansion...

We can study the ρ and σ mesons in Unitarized ChPT for larger quark masses (chiral extrapolation)

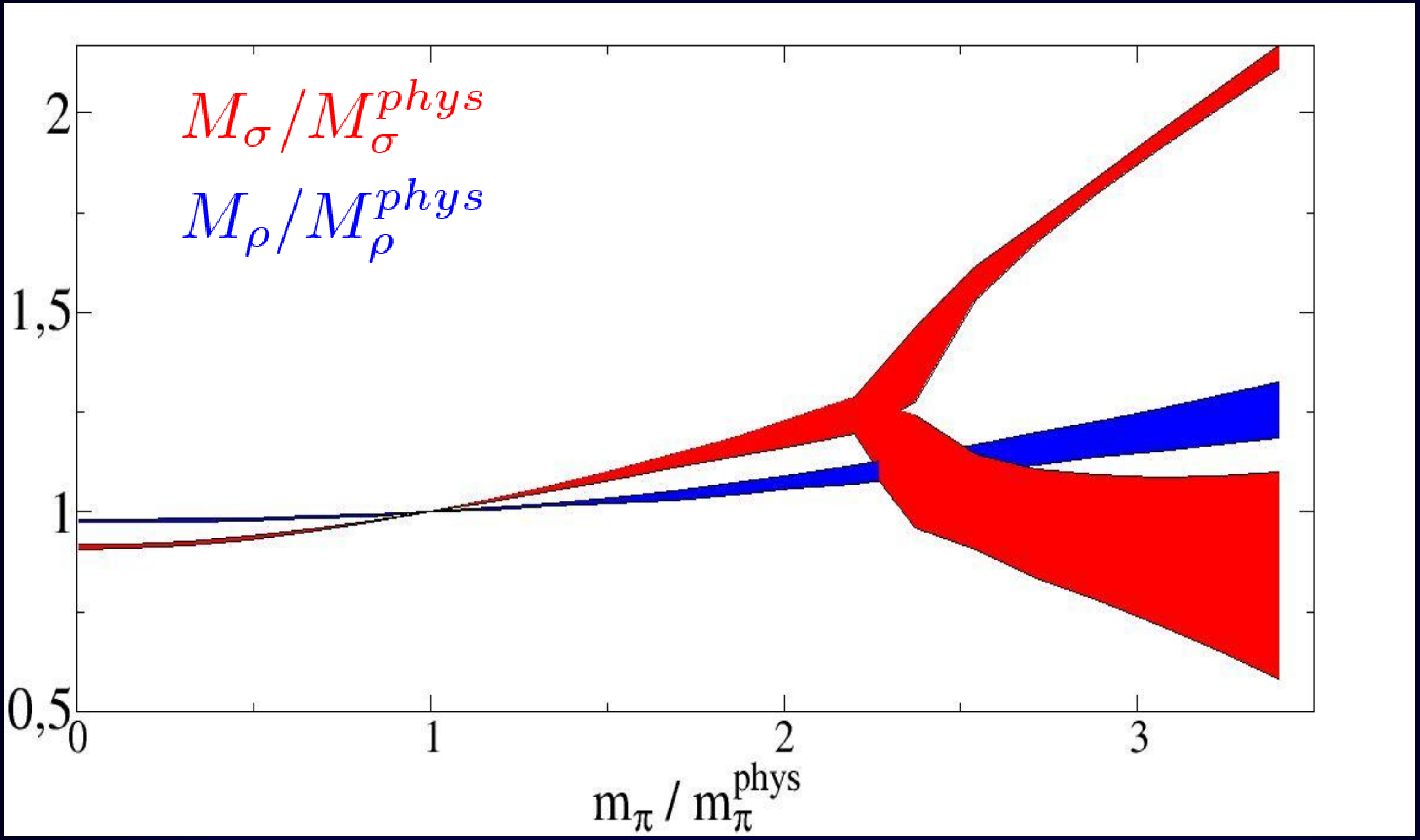
Chiral extrapolation

Change m_π in the amplitudes and see how tho poles evolve



Chiral extrapolation

Resonance mass m_π dependence



There is a "non-analyticity" in the sigma m_π dependence.

The rho mass grows slower than sigma. Both GROW slower than the pion

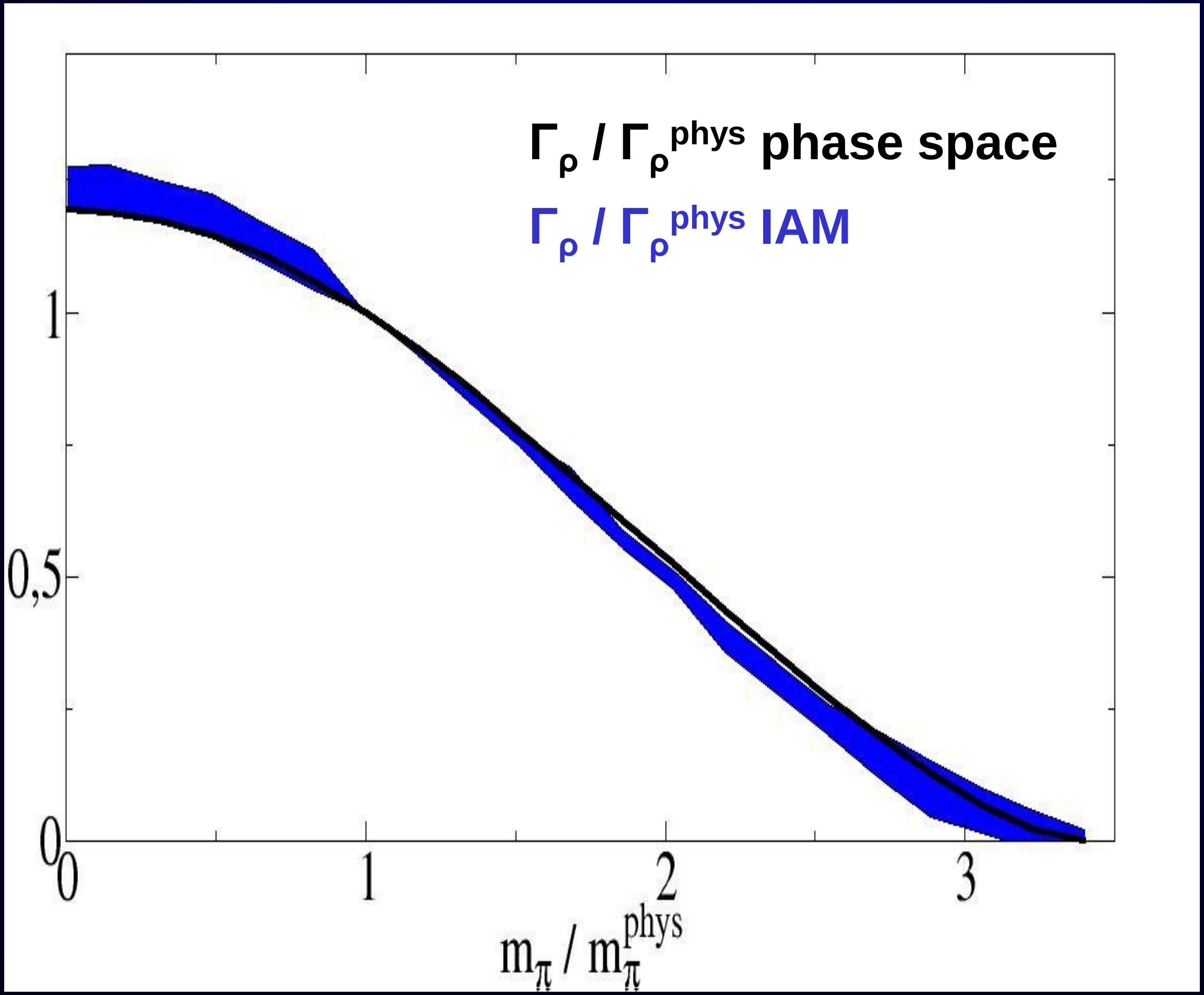
Width behavior comparison with phase space

For a narrow vector particle (like the rho) the decay width is given by

$$\Gamma_\rho = \frac{g^2 |\vec{p}|^3}{6\pi M_\rho^2} \begin{cases} \dfrac{|\vec{p}|^3}{M_\rho^2} & \text{Phase space} \\[2em] g^2 & \text{Coupling to pions} \end{cases}$$

We can calculate the width variation due to phase space reduction and compare with our results. The difference give the dependence of the coupling constant on the pion mass

Width behavior comparison with phase space

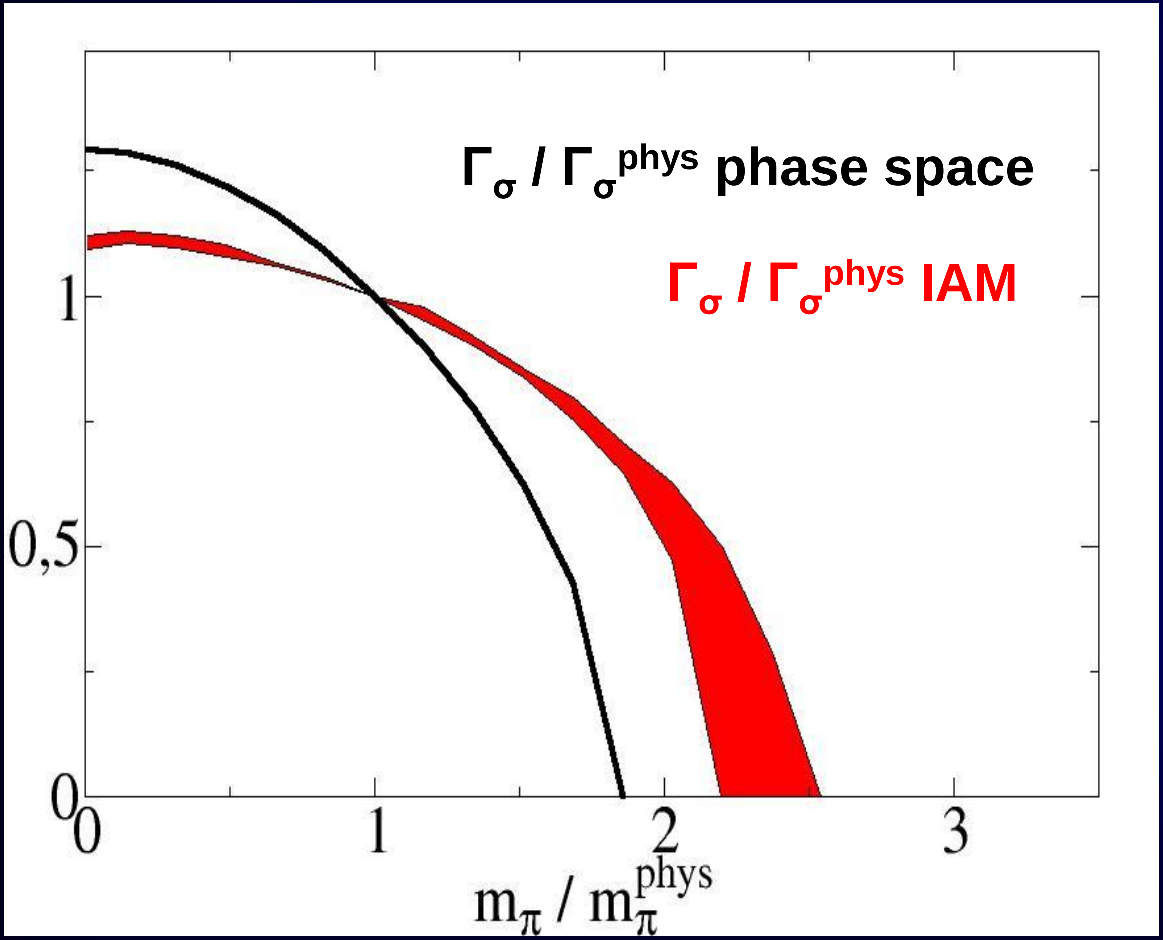


Width behavior explained by phase space

Coupling constant almost independent of m_π

Width behavior comparison with phase space

In contrast, the sigma “width” **does not follow** the decrease in phase space of a Breit-Wigner resonance:



$$\Gamma_\sigma = \frac{g^2 |\vec{p}|}{8\pi M_\sigma}$$

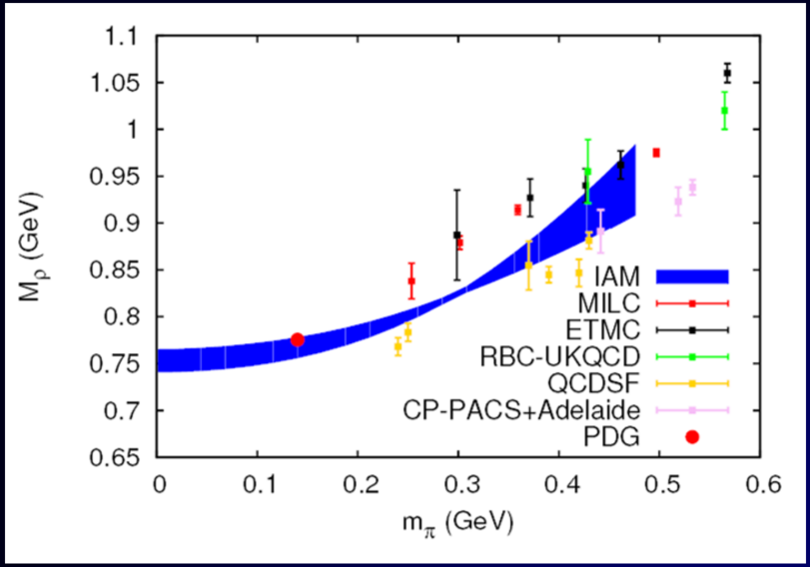
Very bad approximation for a wide resonance as the sigma

Differences with only phase space reduction

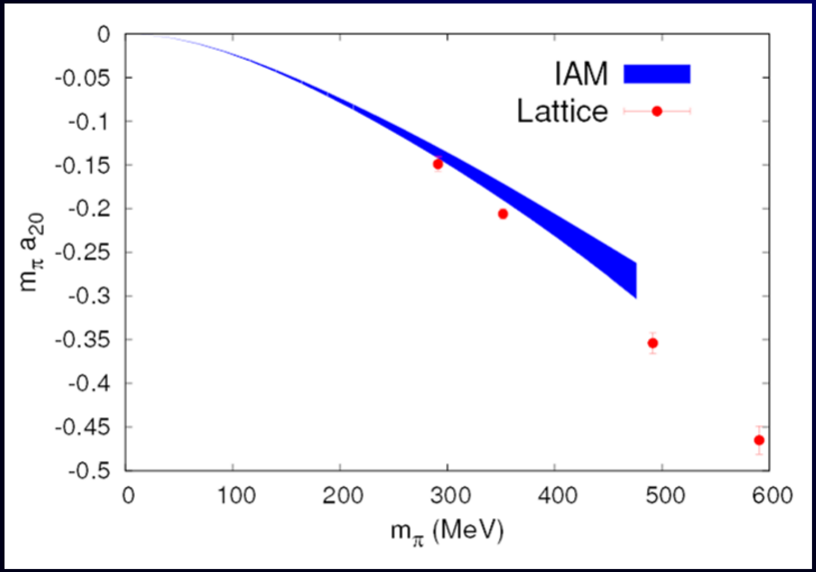
g dependence on m_π

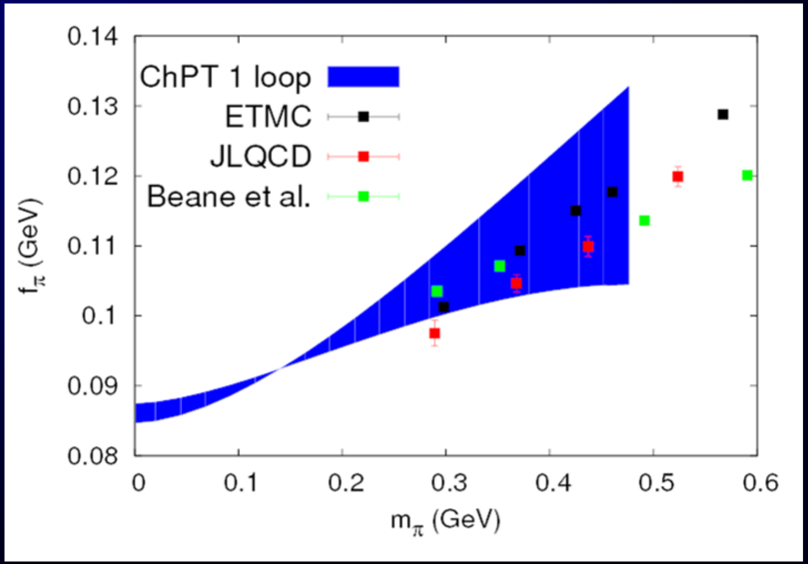
The **dynamics** of the sigma decay depends strongly on the pion-quark mass
Recall that some pion-pion vertices in ChPT depend on the pion mass.

Comparison with lattice



With the LECs from fit to data we find a good agreement with lattice





Summary

Inverse Amplitude Method

Simultaneously resonances and low energy π-π

Generates ρ and σ resonances, without a priori assumptions on their existence or nature

Leading N_c behavior of resonances

qqbar nature of the ρ remarkably well reproduced

σ NOT predominantly quark-antiquark
SUBDOMINANT quark-antiquark component around 1.1 GeV.
(Suggests mixing with heavier ordinary scalar nonet)

Chiral Extrapolation

σ mass dependence on m_π stronger. Non analyticity

ρππ coupling m_π independent. σππ strongly m_π dependent

IAM compares well with lattice results

Thank you!